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Forecasting the Daily Stock Market Volatility of the TASI Index: An ARCH Family Models Approach

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ABSTRACT

Autoregressive conditional heteroscedasticity (ARCH) family models are commonly used to model and forecast the volatility of stock markets around the world. The best-performing model differs, however, depending on the market and time period of interest. Based on twenty years of daily closing data from October 19, 1998, to April 5, 2018, which contains 5,195 trading days, this study seeks to reexamine the ARCH family model that most accurately forecasts the volatility of the Tadawul (i.e., the Saudi stock exchange) All Share Index (TASI). The data set is divided into in-sample and out-of-sample periods. The forecasting models of the study considers range from the simple ARCH and generalized ARCH (GARCH) (1,1) models to more complex models, including exponential GARCH (EGARCH) (1,1) and threshold ARCH (TARCH) (1,1). The study indicates that the EGARCH(1,1) model outperforms all other models in forecasting the short-term volatility of TASI index returns, whereas the GARCH(1,1) model outperforms all other models in forecasting the long-term volatility of TASI index returns.

Keywords: Modeling volatility, forecasting, stock market, ARCH family models, TASI.

JEL classification codes: C53, C58, G17.

1. Introduction

The past decade has been a veritable roller coaster for oil prices, including a great deal of volatility and some sharp declines. One decline, in 2008, was associated with the financial crisis. In response to this event, the Basel Committee of Bank Supervision agreed on an additional Basel Accord (BASEL III) in 2011. BASEL III then introduced these new requirements in 2013 to strengthen bank capital requirements and to decrease leveraging activities. The other decline may still be ongoing: oil prices have fallen from over US\$100 per barrel in 2013 to mid-US\$50 per barrel in 2017, which represents a 50 percent reduction. By the same token Saudi stock equity prices have fallen since 2015, and these moves have generally followed the course of oil prices and this indicates that Saudi stock equity prices are heavily influenced by oil prices. In response to this crash, Saudi Arabia, an oil-based economy, introduced its “vision 2030” program in 2016, an ambitious plan to transform the Saudi economy away from oil dependence. Hence, it is vital to reexamine the ARCH family model that most accurately forecasts the volatility of the Tadawul (i.e., the Saudi stock exchange) All Share Index (TASI)

A number of time-series approaches have been developed to forecast the volatility of stock market indices. Financial index volatility can be defined as the degree by which an index value fluctuates around its average value over a period of time; the standard deviation of returns is a common measure of volatility. Engle and Mezrich (1995) defined volatility as a process that “evolves over time in random but predictable ways.” Stock market volatility is a major concern for financial institutions and investors, economic policy makers, academics, and regulators alike. Busch et al. (2011) have argued that

precise volatility forecasts can benefit several financial applications, including option pricing, asset allocation, and hedging. In addition, Lee et al. (2002) discussed the idea that broad market volatility reflects investor sentiment, which is correlated with both business investment and aggregate consumption and may be loosely tied to economic business cycles.

The aim of the present study is to reexamine which model from the autoregressive conditional heteroscedasticity (ARCH) family of models can provide the most accurate volatility forecasts. More specifically, the performance of the standard ARCH model proposed by Engle (1982) will be compared to a number of ARCH models with additional components such as the generalized ARCH (GARCH) model, which was independently developed by Bollerslev (1986) and Taylor (1986), the threshold ARCH (TARCH) model from Brooks (2008), and the exponential generalized ARCH (EGARCH) model introduced by Nelson (1991). Model accuracy is based on in-sample statistical performance.

The current study uses daily data from TASI, the major index in Saudi Arabia that is supervised by the Capital Market Authority (CMA). The index in 2017 included 171 publicly traded local companies. Although quite a few studies have utilized ARCH family models to forecast stock index volatility, few have compared forecasting power across models, particularly in the Saudi context. A study of this kind might provide useful information on the most appropriate model to use when forecasting the volatility of the TASI. The most precise forecast resulting from this study can substantially influence financial applications and provide insight into the future riskiness of Saudi financial markets.

2. Literature Review

An abundance of empirical work has been conducted to forecast the volatility of stock market returns, although the majority of this work has focused on foreign stock indices. Because these markets are likely to perform differently from the Saudi stock market, any findings from these studies cannot be directly applied to TASI. Ng and McAleer (2004) compared the predictive forecasting power of Bollerslev's (1986) GARCH(1,1) model and an asymmetry-accommodating GJR(1,1) model introduced in 1993 by Glosten, Jagannathan, and Runkle for both the S&P 500 index and the Nikkei 225 index. (The GJR model, which is similar to the TARARCH model, includes leverage terms for modeling asymmetric volatility clustering.) Ng and McAleer's (2004) empirical results indicated that the forecasting performance of each model was dependent on the data used; the authors also found that the GJR(1,1) appeared to perform better with S&P 500 data, whereas the GARCH(1,1) model showed greater predictive power with Nikkei 225 data.

Because the volatility of stock indices exhibit time-varying properties, it is important to frequently estimate and compare the forecasting performance of the conditional volatility models. Alam et al. (2013) evaluated the performance of five ARCH family models when forecasting the volatility of two Bangladesh stock indices, the DSE20 and the DSE general. The authors found that past volatility significantly influenced future volatility in both indices when using any of the five ARCH family models; they also found that the EGARCH model indicated the presence of asymmetric behavior in volatility. Alam et al. (2013) evaluated each model based on both in-sample and out-of-sample statistical and trading performance. In their study, statistical performance was based on the

mean absolute error, mean absolute percentage error, root mean squared error, and their inequality coefficient, while trading performance was based on annualized returns, annualized volatility, the Sharpe ratio, and the maximum drawdown. Although the study provided a strong framework for evaluating the performance of the conditional volatility models, its results (in the case of both the DSE20 and DSE general) were rather inconclusive. In addition, an overall lack of explanation throughout the study and numerous technical errors have left ample room for improvement.

AL-Najjar (2016) concentrated on Jordan's Stock Market Volatility Using ARCH and GARCH Models. Her study applied; ARCH, GARCH, and EGARCH to investigate the behavior of stock return volatility for Amman Stock Exchange (ASE) covering the period from Jan. 1 2005 through Dec.31 2014. Her findings suggest that the symmetric ARCH and GARCH models can capture characteristics of ASE, and provide more evidence for both volatility clustering and leptokurtic, whereas EGARCH output reveals no support for the existence of leverage effect in the stock returns at Amman Stock Exchange. Similarly Mhmoud and Dawalbait (2015) evaluated the forecasting performance of several conditional volatility models. Their study used daily data from Saudi Arabia's TASI index returns over an approximately twelve-year span. The forecasting models considered in their study ranged from the relatively simple GARCH(1,1) model to more complex GARCH models, including the EGARCH(1,1) and GRJ-GARCH(1,1) models. Their forecasting evaluation was based on two sets of statistical criteria for the six-month out-of-sample forecast period. First, to select the volatility model that most closely followed the conditional variance of the return series, Mhmoud and Dawalbait

(2015) used the Ljung-Box Q statistics on both the standardized and squared standardized residuals as well as the Lagrange multiplier (ARCH-LM) test. The authors also used several other information criteria, including the Akaike information criteria (AIC) and maximum log-likelihood (LL) values, to determine the most appropriate model. Mhmoud and Dawalbait (2015) included a statistical evaluation of out-of-sample performance, similarly to Alam et al.'s (2013) study. Statistical performance was again based on four forecast error statistics: the mean absolute error, the mean absolute percentage error, the root mean squared error, and the Theil-U statistic.

Using the two information criteria (AIC and the maximum log-likelihood values) as selection criteria, Mhmoud and Dawalbait (2015) found that the GRJ-GARCH(1,1) model was the best model using the AIC, although they considered the EGARCH(1,1) model to be the best model when using the maximum log-likelihood value. Although Mhmoud and Dawalbait (2015) only nominated the EGARCH(1,1) model once as the best model when using the information criterion, they suggested that this model would outperform the simple GARCH(1,1) model in the presence of asymmetric responses to economic and financial shocks. For statistical forecasting performance, their study found that the GRJ-GARCH(1,1) model outperformed all other models in forecasting the volatility of the TASI index.

Mhmoud and Dawalbait's (2015) model evaluation approach is somewhat different from the approach used in Alam et al.'s study of Bangladesh's stock indices. Although both studies included identical statistical forecast performance evaluations, Mhmoud and Dawalbait (2015) provided additional evaluations through a set of information criteria, while Alam et al.

(2013) expanded the evaluation criteria to include trading performance measures. Mhmoud and Dawalbait's model selection process may have benefited from the inclusion of similar out-of-sample trading performance evaluations; their study could also be enhanced by expanding its data set. The study used daily data from the period January 1, 2005, to December 31, 2012. Of a total of 2,317 observations, Mhmoud and Dawalbait (2015) used the final 124 observations to produce an out-of-sample forecast. Their decision to examine this seven-year period, however, was not justified within their study. With a plethora of additional TASI index data now available, the study could benefit from expanding its data horizon.

Stock markets are highly volatile, which makes modeling these index returns fairly difficult. But having accurate volatility forecasts is incredibly valuable in the financial industry, and such forecasts can benefit a number of financial applications. These forecasts are also valuable to policy makers and academics interested in understanding stock market dynamics. The present study determines the most accurate model for forecasting the volatility of the TASI based on statistical and trading performance. A few past studies that have examined stock market indices including the TASI have utilized a relatively small and unjustified range of observations. The current study includes daily TASI closing data starting from the inception of the index in October 1998. This study will benefit financial engineers and researchers by defining the volatility model that is most appropriate to underlie a number of prominent financial applications.

3. Methodology

This study seeks to reexamine which model from the ARCH family of models can provide the most accurate volatility forecasts. The first model to provide a framework for volatility modeling (included as a model of interest in this study) was the ARCH model proposed by Engle (1982). The ARCH(1) model, which the present study includes as a benchmark model, suggests that the variance of the residuals in the current period are dependent on the squared error terms from the previous period. The model is provided by equation (1).

$$\sigma_t^2 = \alpha_0 + \alpha_1(u_{t-1})^2 \quad (1)$$

Where σ_t^2 is the conditional variance, α_0 and α_1 are constant term and u_t is the error generated from the mean equation at time t. Bollerslev (1986) later introduced the GARCH model, which expanded on the work of Engle (1982). Bollerslev suggested that the variance of the residuals in the current period is dependent on both the squared error terms from the previous period and the past variance. The GARCH(1,1) model is expressed in equation (2).

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta(\sigma_{t-1})^2 \quad (2)$$

A number of previous empirical studies have found that stock market index returns exhibit asymmetries—that is, negative financial and economic shocks tend to increase volatility more than positive shocks do.

The GARCH(1,1) model is symmetric and is incapable of capturing the asymmetric behavior that is common across most stock market return data. The present study includes several extensions of the GARCH model, including the EGARCH(1,1) model developed by Nelson (2001) and the TARCH(1,1) model from Brooks (2008), to account for potential asymmetric behavior. The TARCH

model factors in the asymmetric behavior of the return series and includes a multiplicative dummy variable to determine whether a statistically significant difference occurs when shocks are negative. The TAR(1,1) process is represented in equation (3).

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta \sigma_{t-1}^2 + \gamma u_{t-1}^2 I_{t-1} \quad (3)$$

where I_{t-1} is a dummy variable that is equal to 1 when a shock is negative and 0 when a shock is positive. This model postulates that “good” and “bad” news have differential effects on the conditional variance. If bad news increases volatility, then a leverage effect is present, and the response to a given shock is not symmetric.

Similarly, the EGARCH model is based on a logarithmic conditional variance expression that can be written as in equation: (4).

$$\ln \sigma_t^2 = \omega + \beta \ln \sigma_{t-1}^2 + \gamma \frac{u_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \alpha \left[\frac{|u_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}} \right] \quad (4)$$

This equation illustrates the asymmetric responses in volatility to past shocks. The log of the conditional variance implies that the leverage effect is exponential and guarantees that forecasts of the conditional variance are nonnegative.

4. Data Analysis

The data used in this study consist of daily TASI performance observations collected from the Tadawul database. In this study, the daily observations were drawn from the index closing price. These data were collected from October 19, 1998, to April 5, 2018, which contains 5,195 trading days. The total data were

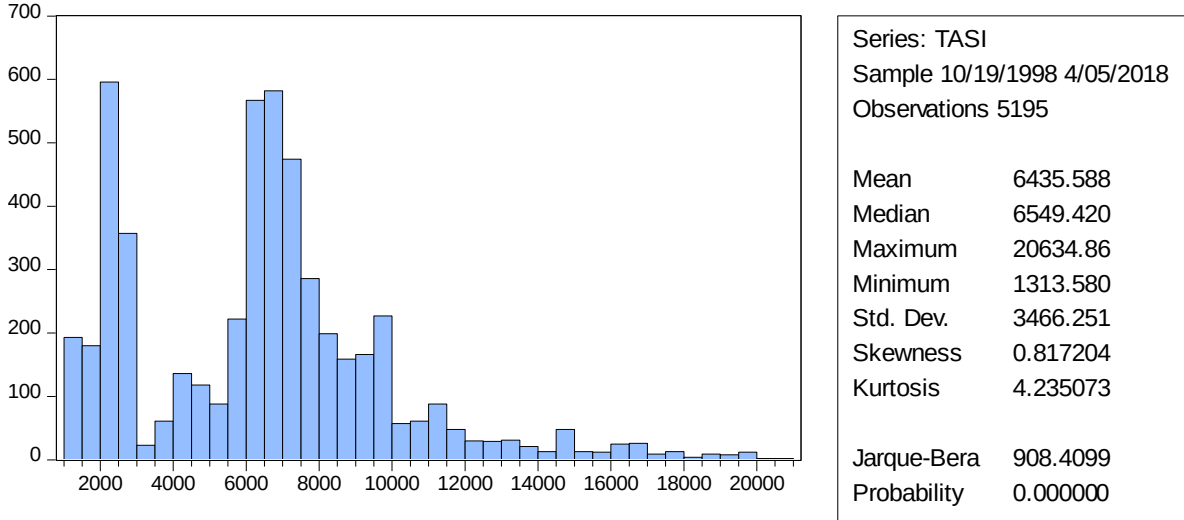
divided into in-sample and out-of-sample data sets for forecasting purposes. Both one-month-ahead (short-term) and one-year-ahead (long-term) periods were estimated and evaluated. The TASI, which includes 171 leading companies in prominent industries of the Saudi economy, is representative of the Saudi equities market and consequently of the Saudi stock market.

A number of pre-estimation tests must be performed prior to modeling and forecasting the conditional variance of the TASI, as discussed in the following sections.

4.1 Jarque-Bera Statistic

The present study first examined the normality of TASI. The histogram and summary statistics presented in figure 1 indicate that the TASI has a positive skew (0.81) and high kurtosis (4.23) when compared to a normal distribution. The Jarque-Bera statistic reveals that the index is non-normal, at a 1 percent significance level. This result provides reason to convert the TASI series into a return series.

Figure 1. Histogram and Summary Statistics of TASI index



4.2 Transformation of TASI Series

Generally, stock indices are non-stationary and rather unpredictable over time. This measure is not appropriate for time-series analyses. The continuous compounding of TASI series (P_t) converts this series into a return series (R_t) using equation (5).

$$R_t = \ln \frac{P_{t+1}}{P_t} = \ln P_{t+1} - \ln P_t \quad (5)$$

Where P_t and P_{t+1} are the closing prices for two consecutive periods. The logarithmic difference is symmetric for positive and negative movements and is expressed in percentage terms.

4.3 Descriptive Statistics and Stationarity Checks on Tests on TASI Returns

Table 1 shows that the mean of the TASI return series is close to zero, which was expected. The standard deviation of the series is relatively high, indicating

that daily returns fluctuate sizably. The negative skewness of the return series also indicates that the asymmetric tail of the distribution extends toward negative values. The return series also has a large, positive kurtosis that indicates that the return distribution is fat-tailed. Overall, the return series is non-normal, given the reported Jarque-Bera statistic of 113019.2 with an associated p-value of 0.00. Table 1 also reports both the augmented Dickey-Fuller and Phillips-Perron tests of series stationarity. Both tests reject the null hypothesis of series non-stationarity at the 1 percent level of significance.

Table 1. TASI index returns

Descriptive statistics	
Mean	0.000321
Standard deviation	0.013948
Skewness	-0.894215
Kurtosis	13.75784
Jarque-Bera	25738.34
Prob. value	<0.001
Augmented Dickey-Fuller	-39.39605***
Phillips-Perron	-66.80958***
Sample size	5,194

*** Indicates significance at the 1% level

4.4 Conditional Mean Specification and ARCH LM Test

In order to estimate and forecast the conditional variance for a series, a conditional mean equation must first be specified. In this study, an autoregressive moving average or ARMA(1,1) process was selected to model the conditional mean. This selection is based on both the Akaike and Schwarz information criteria. These information criteria were almost identical across ARMA(1,1), autoregressive or AR(1), and moving average or MA(1) models. The ARCH Lagrange multiplier (LM) test was then used to check for volatility clustering and heteroscedasticity in the data series. The null hypothesis for the ARCH LM test suggests that no ARCH effects are present. The ARCH LM test reports an F-statistic of 0.07 with an associated p-value of 0.78. Therefore, the TASI return has no clustered volatilities nor heteroscedasticity.

5. Results and Discussion

Tables 2 through 5 show the estimates of the ARCH(1), GARCH(1,1), TARARCH(1,1,1), and EGARCH(1,1) models for daily TASI returns. The first set of outputs reported in the tables includes TASI returns data from October 19, 1998, to April 5, 2018. These estimates were used to forecast short-term, one-month-ahead series volatility. The second set of outputs reported in the tables includes TASI returns data from October 19, 1998, to May 5, 2017. These estimates were used to forecast long-term, one-year-ahead series volatility. All estimations assume a Student's-t distribution, which previous studies have shown to be a more appropriate distribution when managing a large sample.

5.1 Output of ARCH Family Models on TASI Returns

The outputs of both ARCH model estimates for the TASI returns indicate that all terms in the conditional mean and variance equations were statistically significant at the one percent level. An important result from these outputs is that the squared error terms from the previous period (α_1) are significant in the variance equation and have a significant impact on current volatility.

Table 2. Estimates of the ARCH(1) model

$R_t = a_0 + a_1 R_{t-1} + a_2 u_{t-1} + u_t$ $\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2$				
a_0	a_1	a_2	α_0	α_1
0.0008 (5.97)***	-0.21 (2.73)**	0.13 (1.25)**	0.00016 (4.31)***	1.671 (4.05)***
Akaike information criterion: 6.349941 Schwarz information criterion: 6.342368 Log-likelihood: -16493.62				
a_0	a_1	a_2	α_0	α_1
0.0167 (4.15)***	-0.19 (-4.87)***	0.09 (3.378)***	0.0064 (6.78)***	1.534 (8.547)***
Akaike information criterion: 6.154575 Schwarz information criterion: 6.154341 Log-likelihood: -16478.52				

Values in parentheses are z-statistics;

*** indicates significance at the 1% level; ** indicates significance at the 5% level.

Table 3. Estimates of the GARCH(1,1) model

$R_t = a_0 + a_1 R_{t-1} + a_2 u_{t-1} + u_t$ $\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta \sigma_{t-1}^2$	
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a_0	a_1	a_2	α_0	α_1	β
0.00747 (5.051)***	-0.1667 (-1.368)	0.068 (2.34)***	0.254 (4.98)***	0.515 (13.35)***	0.2214 (25.15)***

Akaike information criterion: 6.518202
Schwarz information criterion: 6.509366
Log-likelihood: -16931.51

a_0	a_1	a_2	α_0	α_1	β
0.0454 (6.512)***	-0.187 (-2.71)**	0.0458 (5.61)***	0.2487 (5.12)***	0.658 (9.124)***	0.1973 (25.12)***

Akaike information criterion: 6.51121
Schwarz information criterion: 6.50156
Log-likelihood: -11027.1

Values in parentheses are z-statistics;

*** indicates significance at the 1% level; ** indicates significance at the 5% level.

The GARCH model outputs indicate that the AR component of the conditional mean equation was not significant in the estimates for short-run forecasting purposes but was significant at the 5 percent level in the estimates for long-run forecasting purposes. In both estimations, all GARCH model conditional variance components were statistically significant at the one percent level, which means that current volatility is influenced by squared error terms from the previous period (α_1) and past volatility (β).

Table 4. Estimates of the TAR(1,1,1) model

$R_t = a_0 + a_1 R_{t-1} + a_2 u_{t-1} + u_t$ $\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta \sigma_{t-1}^2 + \gamma u_{t-1}^2 I_{t-1}$						
a_0	a_1	a_2	α_0	α_1	β	γ
0.3173 (3.15)***	-0.5 (-2)	0.781 (1.987)***	0.0124 (6.561)***	0.0154 (6.451)***	0.1754 (37.154)***	0.1542 (9.8113)***

Akaike information criterion: 4.154721
Schwarz information criterion: 4.034513
Log-likelihood: -13457.17

α_0	α_1	α_2	α_0	α_1	β	γ
0.0157 (6.543)***	-0.50 (-2.1)	0.9345 (2.932)***	0.01574 (8.754)***	0.01365 (6.324)***	0.1274 (55.32)***	0.0157 (25.34)***

Akaike information criterion: 4.156421

Schwarz information criterion: 4.024513

Log-likelihood: -13450.37

Values in parentheses are z-statistics; *** indicates significance at the 1% level; ** indicates significance at the 5% level.

The TARARCH model outputs illustrate that the AR component of the conditional mean equation was not significant in either estimate. In both runs, all TARARCH model conditional variance components were statistically significant at the one percent level, which means that current volatility is influenced by squared error terms from the previous period (α_1) and past volatility (β) and is also asymmetric in nature (γ).

Table 5. Estimates of the EGARCH(1,1) model

$R_t = a_0 + a_1 R_{t-1} + a_2 u_{t-1} + u_t$ $\ln \sigma_t^2 = \omega + \beta \ln \sigma_{t-1}^2 + \gamma \frac{u_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \alpha \left[\frac{ u_{t-1} }{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}} \right]$						
a_0	a_1	a_2	ω	α	β	γ
0.1673 (6.754)** *	-0.5478 (-1.875)	0.986 (2.75)**	-0.0875 (-13.1)***	0.3573 (24.2)***	0.00154 (94.2)***	-0.157 (-8.65)***
Akaike information criterion: 5.654213 Schwarz information criterion: 5.554321 Log-likelihood: -135431.15						
a_0	a_1	a_2	ω	α	β	γ
0.1671 (5)***	-0.5034 (-1.564)	0.8921 (2.6)***	-0.0764 (-12.5)***	0.3551 (23.4)***	0.00143 (90.1)***	-0.1431 (-6.54)***
Akaike information criterion: 5.642431 Schwarz information criterion: 5.554215 Log-likelihood: -13541.37						

Values in parentheses are z-statistics;

*** indicates significance at the 1% level; ** indicates significance at the 5% level.

Again, the EGARCH model outcomes illustrate that the AR component of the conditional mean equation was not significant in either estimate. In both estimates, all EGARCH model conditional variance components were statistically significant at the one percent level, which indicates that current volatility is dependent on yesterday's residuals and volatility, and an asymmetric behavior in volatility was present. This implies that "bad" news has a greater impact on TASI returns than "good" news.

5.2 Statistical Performance

The models of interest were then compared in terms of their short-run and long-run forecasting abilities. The evaluation was based on four forecast error statistics: root mean squared error (RMSE), mean absolute error (MAE), mean absolute percent error (MAPE), and the Theil inequality coefficient (TIC). These statistics were computed as follows:

$$\begin{aligned}
 RMSE &= \sqrt{\frac{1}{n} \sum_{t=1}^n (\hat{\sigma}_t - \sigma_t)^2} & MAPE &= \frac{1}{n} \sum_{t=1}^n \left| \frac{(\hat{\sigma}_t - \sigma_t)}{\sigma_t} \right| \\
 MAE &= \frac{1}{n} \sum_{t=1}^n |\hat{\sigma}_t - \sigma_t| & TIC &= \frac{\sqrt{\frac{1}{n} \sum (\sigma_t - \hat{\sigma}_t)^2}}{\sqrt{\frac{1}{n} \sum \hat{\sigma}_t} + \sqrt{\frac{1}{n} \sum \sigma_t}}
 \end{aligned}$$

In all the above statistics, “n” represents the number of in-sample forecasts (one-month-ahead) and one-year-ahead forecasts; σ_i represents the actual volatility experiences at time “t,” while $\hat{\sigma}_i$ is the forecasted volatility at time “t.” Each statistic is calculated by examining the difference between the forecasted conditional variance and their true values. The model that exhibits the lowest values of these error measurements is thus considered to be the best model. Tables 6 and 7 present comparisons of the in-sample statistical performance results for short-term (one-month-ahead) and long-term (one-year-ahead) forecasts, respectively.

Table 6. One-month ahead in-sample statistical performance results for TASI index returns

Performance	Models			
Indicator	ARCH(1)	GARCH(1,1)	TARCH(1,1,1)	EGARCH(1,1)
Root mean squared error	0.981405	0.982033	0.983029	0.982406
Mean absolute error	0.818495	0.819382	0.817891	0.816968
Mean absolute percentage error	170.8819	191.1239	155.4313	140.632
Theil inequality coefficient	1.286674	1.281805	1.2936	1.297988

Note: **bold** text indicates the best performer.

As table 6 shows, the short-term forecast performance results indicate that the ARCH(1) model has the lowest RMSE, at 0.978975; the EGARCH(1,1) model has the lowest MAE and MAPE, at 0.814538 and 140.8644, respectively; finally, the GARCH(1,1) model has the lowest reported Theil inequality coefficient, at 0.909375. The EGARCH(1,1) model outperformed all other models in forecasting the short-term volatility of TASI index returns. These results indicate that a model that includes asymmetry-accommodating parameters is most appropriate for short-term forecasting purposes.

Table 7. One-year-ahead in-sample statistical performance results for TASI index returns

Performance Indicator	Models			
	ARCH(1)	GARCH(1,1)	TARCH(1,1,1)	EGARCH(1,1)
Root mean squared error	1.561076	1.55723	1.561926	1.561681
Mean absolute error	1.39756	1.393416	1.39809	1.397947
Mean absolute percentage error	87.9303	97.8412	83.6473	76.0526
Theil inequality coefficient	1.733593	1.728196	1.735496	1.739934

Note: **bold** text indicates the best performer.

As table 7 shows, the long-term forecast performance results indicate that the GARCH(1,1) model had the lowest RMSE (1.55723), MAE (1.393416), and Theil inequality coefficient (1.728196). The TARCH(1,1,1) model had the lowest reported MAPE, at 83.6473. The GARCH(1,1) model had the lowest error measurement values in three of four categories and therefore outperformed all other models in forecasting the long-term volatility of TASI index returns. These results show that the relatively simple symmetric GARCH model performs better in forecasting longer-term, one-year-ahead

conditional variance of the TASI index returns, despite the existence of asymmetries in the data.

These results indicate that including “leverage effects” or asymmetric components is important for forecasting short-run volatility. When the forecast horizon is expanded, however, the inclusion of asymmetric components does not benefit conditional volatility forecast performance, and simpler ARCH family models perform better than those that are more complex.

6. Conclusion

This study has employed the ARCH family model approach to forecast the daily stock market volatility of the TASI index. The data were collected from October 19, 1998, to April 5, 2018, which represents 5,195 trading days. The data set was divided into in-sample and out-of-sample periods. The forecasting models considered in this study ranged from the simple ARCH and GARCH(1,1) models to more complex models, including EGARCH(1,1) and TARARCH(1,1). Models were selected based on out-of-sample statistical performance.

This study has indicated that the EGARCH(1,1) model outperforms all other models in forecasting the short-term volatility of TASI index returns. The inclusion of leverage effects or asymmetric components is thus important for forecasting short-run volatility. In addition, the GARCH(1,1) model was shown to outperform all other models in forecasting the long-term volatility of TASI index returns. When the forecast horizon is expanded, the inclusion of asymmetric components thus does not benefit conditional volatility forecast

performance; simpler ARCH family models perform better than those that are more complex.

Based on the results, a promising next step that should be undertaken to further advance the research presented in this study would be to identify the structural break in the series mean and variance using the Pruned Exact Linear Time (PELT) algorithm, where structural breaks dates are captured using dummy variables in the GARCH models.

7. References

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